MSRI

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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: KARA KOZIOL Email/Phone: kkoziol@ualberta.ca

Speaker's Name: ANA CARAIANI

Talk Title: MODULI OF REPRESENTATIONS AND GLOBAL LANGLANDS PARAMETRIZATION I

Date: 4/10/19 Time: 10:30 am/ pm (circle one)

Please summarize the lecture in 5 or fewer sentences: THE SPEAKER BEGAN DISCUSSING MODULI OF GALOIS REPS APPEARING IN LAFFORGUE'S RESULT, NAMELY HOW EXCURSION OPERATORS GIVE RISE TO PSEUDO REPRESENTATIONS, AND THUS SEMI SIMPLE GALOIS PARAMETERS

CHECK LIST

(This is **NOT** optional, we will **not** pay for **incomplete** forms)

☒ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.

☒ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.

- **Computer Presentations**: Obtain a copy of their presentation
- **Overhead**: Obtain a copy or use the originals and scan them
- **Blackboard**: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
- **Handouts**: Obtain copies of and scan all handouts

☒ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.

☒ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)

☒ Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

MODULI OF REPRESENTATIONS + GLOBAL LANGLANDS PARAMETRIZATION I — CARAIANI

§1 OVERVIEW

- $G/\mathbb{F}_q$ SPLIT CONNECTED RED'IVE
- $F = \mathbb{F}_q(X)$
- $E/\mathbb{Q}_\ell$ FINITE
- $\widehat{G}$ LANGLANDS DUAL OF G OVER E
- $\mathrm{REP}(\widehat{G})$ = F.D ALG REPS OF $\widehat{G}$ OVER E
- $\Gamma_F = \mathrm{GAL}(\overline{F}/F)$ w/ PROFINITE TOP
- $\mathrm{REP}(\Gamma_F)$ CTS FINITE DIM'L REPS OF Γ_F OVER E
- WE'VE CONSTRUCTED A SYSTEM OF E-LINEAR FUNCTORS INDEXED BY I (w/ CERTAIN COMPAT. AS I VARIES)

$$H_I \quad \mathrm{REP}(\widehat{G}^I) \longrightarrow \mathrm{REP}_{(\pi_N)}(\Gamma_F^I)$$
$$W \longmapsto H_I(W) := H_I(W)^{\mathrm{cusp}}$$

EG $\quad H_\emptyset(\mathbb{1}) = C_c^{\mathrm{cusp}}(G(F)\backslash G(\mathbb{A})/K_N\Xi, E)$

COMPATIBILITIES:

$$\xi : I \longrightarrow J \quad \rightsquigarrow \quad \hat{G}^J \longrightarrow \hat{G}^I$$

$$(g_j)_{j \in J} \longmapsto (g_{\xi(i)})_{i \in I}$$

$$\Gamma_F^J \longrightarrow \Gamma_F^I$$

$\rightsquigarrow$ RESTRICTION FUNCTORS

$$\mathrm{Rep}(\hat{G}^I) \longrightarrow \mathrm{Rep}(\hat{G}^J) : W \longmapsto W^{\xi}$$

$$\mathrm{Rep}(\Gamma_F^I) \longrightarrow \mathrm{Rep}(\Gamma_F^J)$$

$\exists$ ISOM $\quad \chi_\xi : H_I(W) \xrightarrow{\sim} H_J(W^\xi)$

Γ_F^J-EQUIV (ACTION ON LHS VIA $\Gamma_F^J \longrightarrow \Gamma_F^I$)

χ_ξ IS FUNCTORIAL IN W, COMPATIBLE W/ COMPOSITION

$\boxed{\begin{array}{l}\text{THM (V. LAFFORGUE) } \forall I \text{ FINITE, WE HAVE A } \oplus \text{ DECOMP} \\ \quad H_I(W) = \bigoplus_\sigma H_I(W)_\sigma, \quad \text{WHERE } \sigma : \Gamma_F \longrightarrow \hat{G}(\overline{\mathbb{Q}}_\ell) \\ \text{RUNS OVER A FINITE SET OF SEMISIMPLE REPS}\end{array}}$

THESE σ SATISFY THESE EXTRA CONDITIONS:

1) σ IS CTS AND VALUED IN A FINITE EXT E'/E

2) σ IS UNRAMIFIED OVER $X \setminus N$

3) AT $v \in X \setminus N$, σ SATISFIES FROBENIUS - HECKE COMPATIBILITY

IE, $\forall V \in \mathrm{IRR}(\hat{G}) \rightsquigarrow h_{V,v} \in C_c(K_v \backslash G(F_v)/K_v, E)$

$$\underset{\Pi_N}{\overset{\cap}{T(h_{V,v})}} \quad \text{AND} \quad \nu(T(h_{V,v})) = \text{EIGENVALUE OF } T(h_{V,v}) \text{ ON } H_I(W)_\sigma$$

$$= \chi_V(\sigma(\mathrm{FROB}_v))$$

THE KEY INGREDIENTS IN PROOF

- EXTEND Π_N-ACTION ON $H_I(W)$ TO AN ACTION OF COMM. ALG. B OF "EXCURSION OPERATORS" (NEED TO DO THIS FOR GENERAL GROUPS G) TO DETERMINE σ

- VERSION OF KOTTWITZ CONJECTURE SUGGESTS THAT ACTION SHOULD EXTEND TO B

- $\nu : B \longrightarrow \overline{\mathbb{Q}}_\ell \rightsquigarrow$ PSEUDOREP OF Γ_F VALUED IN $\hat{G}(\overline{\mathbb{Q}}_\ell)$

- GO FROM PSEUDOREP TO SEMISIMPLE REP $\sigma : \Gamma_F \longrightarrow \hat{G}(\overline{\mathbb{Q}}_\ell)$ UP TO $\hat{G}$-CONJ

NEW TODAY

IDEA OF DRINFELD : $(H_I)_I$ GIVES RISE ~~OF~~ TO A QCOH SHEAF ON MODULI STACK OF $\widehat{G}$-VALUED REPS OF Γ_F

WEIDNER : MORE CONCISE DEF OF PSEUDO REP OF $\widehat{G}$-VALUED REPS

(SEE NOT-SO-SECRET NOTES OF ZHU)

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§2 EXCURSION OPERATORS

FOCUS ON B-ACTION ON $H_\emptyset(\mathbb{1})$

CHOOSE $I \neq \emptyset$, AND • $(\gamma_i)_{i \in I} \in \Gamma_F^I$

$\longleftrightarrow$ GRP HOM

$FG(I) \longrightarrow \Gamma_F$

"PROBING Γ_F BY FREE F.G. GRPS"

• $W \in \mathrm{Rep}(\widehat{G}^I)$

$\rightsquigarrow W^\vee$ E-LINEAR DUAL

$x \in W^{\Delta(\widehat{G})}$, $\xi \in (W^\vee)^{\Delta(\widehat{G})}$

$\Updownarrow$

$\mathbb{1} \xrightarrow[\widehat{G}\text{-EQUIV}]{} W^\xi$, $\xi : W^\xi \longrightarrow \mathbb{1}$

$$\begin{tikzcd}[column sep=large]
H_\emptyset(\mathbb{1}) \arrow[r, "\sim"] \arrow[d, "S_{I,W,x,\xi,\gamma_I}"'] & H_{\{*\}}(\mathbb{1}) \arrow[r, "H_{\{*\}}(x)"] \arrow[d, phantom, "\circlearrowright"] & H_{\{*\}}(W) \arrow[r, "\text{``CREATION''}\ \sim", "\chi_\xi"'] & H_I(W) \arrow[d, "\gamma_I", "\text{``ANNIHILATION''}"'] \\
H_\emptyset(\mathbb{1}) & H_{\{*\}}(\mathbb{1}) \arrow[l, "\sim"'] & H_{\{*\}}(W) \arrow[l, "H_{\{*\}}(\xi)"] & H_I(W) \arrow[l, "\sim"', "\chi_\xi"]
\end{tikzcd}$$

LEMMA S_{I,W,x,ξ,γ_I} ONLY DEPENDS ON

- $\gamma_I \in \Gamma_F^I$
- $\tilde{f} : \hat{G}^I \longrightarrow E$

  $(g_i)_{i \in I} \longmapsto \langle \xi, (g_i) . x \rangle \in E$

WILL DENOTE IT $\underline{S_{I,\tilde{f},\gamma_I}}$

PF IDEA

$$\tilde{f} \in E[\hat{G}^I]^{\Delta(a_L(\hat{G})) \times \Delta(a_R(\hat{G}))}$$

LEFT ACTION   RIGHT ACTION

THINK OF THIS AS

$$\tilde{f} : \mathbb{1} \longrightarrow E[\hat{G}^I]^{\Delta(a_R(\hat{G}))} : V^\xi$$

$\Delta(a_L(\hat{G}))$-EQUIV

ALSO HAVE

$$ev : V^\xi \longrightarrow \mathbb{1} \quad \text{EVALUATION AT ID}$$

$\rightsquigarrow$ EXCURSION OPERATOR $S_{I,\tilde{f},\gamma_I}$

TO FINISH, NOTE THAT • SUB $\widehat{G}^I$-REP OF V GEN'D BY $\mathcal{F}$ IS ISOM'C TO A SUBQUOT OF W

• FUNCTORIALITY OF FUSION ISOM IN W $\square$

NOTE
REP IS UNRAMIFIED B/C IMAGE OF CREATION IS INERTIA-INVT

<u>RMK</u> EXCURSION OPERATORS ARE DEFINED IN TERMS OF

$I = I' \cup \{*\}$

$$E[\widehat{G}^{I' \cup \{*\}}]^{\Delta(a_L(\widehat{G})) \times \Delta(a_R(\widehat{G}))} \cong E[\widehat{G}^{I'}]^{\Delta(Ad(\widehat{G}))}$$

$$\tilde{\mathcal{F}}(\underset{\widehat{G}^{I'}}{\underset{\cap}{m}}, \underset{\widehat{G}^{\{*\}}}{\underset{\cap}{g}}) = \mathcal{F}(g^{-1}m)$$

REG FNS ON GIT QUOTIENT OF $\widehat{G}^{I'}$ BY $Ad(\widehat{G})$

THE $S_{I, \tilde{\mathcal{F}}, \gamma_I}$ SATISFY IMPORTANT COMPATIBILITIES NEEDED TO EXTRACT A PSEUDOREP

E.G. NEED COMPATIBILITY W/

$$\begin{tikzcd} FG(I) \arrow[r] & T_F \\ FG(J) \arrow[u] \arrow[ur] & \end{tikzcd}$$

THEN $\quad \nu : \mathcal{B} \longrightarrow \overline{\mathbb{Q}}_\ell$ GIVES

$$\sigma : \Gamma_F \longrightarrow \widehat{G}(\overline{\mathbb{Q}}_\ell), \quad \text{AND}$$

$$\nu(S_{I, f, \gamma_I}) = \tilde{f}(\sigma(\gamma_I))$$

- ANALOGOUS STEPS CONSTRUCT

$$S_{I, f, \gamma_I} \circlearrowright H_J(W)$$