

NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

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Speaker's Name: JARED WENSTEIN

Talk Title: MODULI OF SHUKAS I

Date: 4/8/19 Time: 11:00 am / pm (circle one)

Please summarize the lecture in 5 or fewer sentences: THE SPEAKER DISCUSSED
SOME OF THE MAIN OBJECTS IN THE GEOMETRIC LANGUAGES
PROGRAM: THE STACKS OF G-BUNDLES, THE HECKE STACK,
AND THE STACK OF SHUKAS

CHECK LIST

(This is **NOT** optional, we will **not** pay for incomplete forms)

- Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
 - **Computer Presentations:** Obtain a copy of their presentation
 - **Overhead:** Obtain a copy or use the originals and scan them
 - **Blackboard:** Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
 - **Handouts:** Obtain copies of and scan all handouts
- For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list.
(YYYY.MM.DD.TIME.SpeakerLastName)
- Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

MODULI OF SHIMURA I

- WEINSTEIN

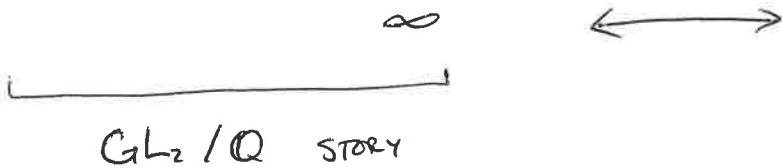
(1)

NUMBER FIELDS

FUNCTION FIELDS $F = k(X)$

- MODULI OF ELLIPTIC CURVES E/\mathbb{Q}
 $E(\mathbb{C}) \cong \mathbb{C}/\Lambda$

- MODULI OF DRINFELD MODULES OVER $A = H^0(X \setminus M, \mathcal{O}_X)$
 \mathbb{C}/Λ



- GL_m/\mathbb{Q}



- CARLITZ MODULES

- CM ELLIPTIC CURVES $/K$
 (GL_1/K) \leftarrow IM QUAD'IC

- MODULI OF DRINFELD SHIMURAS
(ONE SIMPLE ZERO / ONE SIMPLE POLE)

(NO ANALOG... YET)

(L. LAFFORGUE FOR GL_n)

- G -SHIMURAS w/ ARBITRARY LEGS
 \nearrow
RED GP

• $k = \mathbb{F}_q$, X/k GEOM CONN'D, SMOOTH PROJ CURVE

• $F = k(X)$

• G/k SPLIT RED'VE GP

• $\text{BUN}_G =$ STACK WHICH SENDS S/k TO
GROUPOID OF G -BUNDLES ON $X \times_k S$ (= G -TORSORS)

(EX $G = \text{GL}_n$ GIVES LOC. FREE \mathcal{O} -MODS OF RK n)

FOR $G = \text{GL}_n$,

$$\text{BUN}_G(k) = G(F) \backslash G(A) / G(\mathcal{O}), \quad G(\mathcal{O}) = \prod_{v \in |X|} G(\mathcal{O}_v)$$

SKETCH EVERY $\xi \in \text{BUN}_G(k)$ IS TRIVIAL OVER F :

$$\xi: \xi|_U \xrightarrow{\sim} \xi^{\text{TRIV}}|_U \quad U \text{ DENSE OPEN}$$

ALSO OVER EACH \mathcal{O}_v :

$$\xi_v: \xi|_{\mathcal{O}_v} \xrightarrow{\sim} \xi^{\text{TRIV}}|_{\mathcal{O}_v}$$

GET ELEMENT

$$\xi|_{F_v} \circ \xi_v^{-1} \in \text{AUT}(\xi^{\text{TRIV}}|_{F_v}) \cong G(F_v)$$

SO AUT FORMS ARE FMS ON $\text{BUN}_G(k)$

FACT BUN_G IS A SMOOTH ARTIN STACK

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$\Rightarrow \exists$ A SMOOTH UNIFORMIZATION

$$Y \longrightarrow BUN_G$$

SMOOTH, LOCALLY OF FINITE TYPE (BUT NOT OF FINITE TYPE.)
IN GENERAL

EX $BUN_{GL_1} \cong [Pic(X)/G_m] \quad (X(k) \neq \emptyset)$

UNIF'N :

$$\bigsqcup_{d \in \mathbb{Z}} Pic^d(X) \longrightarrow BUN_{GL_1}$$

\uparrow
DEG d LINE BUNDLES

EX $X = \mathbb{P}^1 \quad G = GL_2$

$$BUN_{GL_2} \cong \bigsqcup_{d \in \mathbb{Z}} BUN_{GL_2}^d \quad \curvearrowright \text{DEG } d \text{ V.B.'S}$$

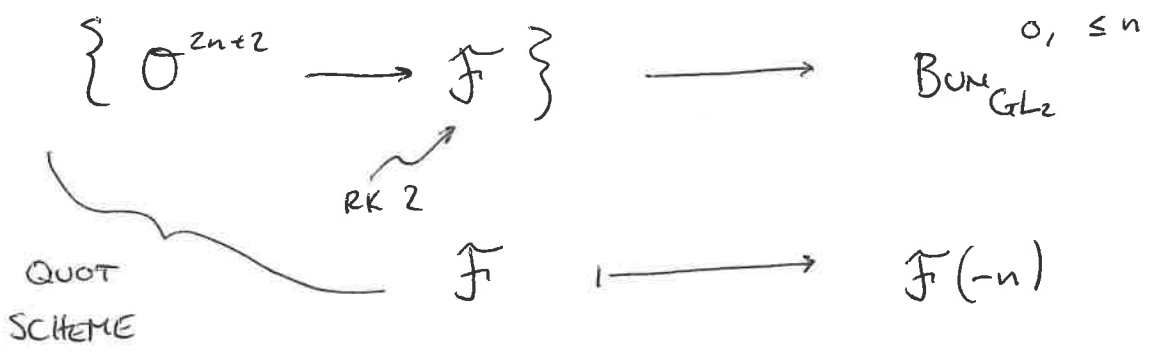
$\mathcal{E} \in BUN_{GL_2}^0(S)$ ARE EVERYWHERE ISOM'IC TO
 $\mathcal{O}(n) \oplus \mathcal{O}(-n), \quad n \geq 0$

DEFINE

$$BUN_{GL_2}^{0, \leq n} = \left\{ \mathcal{E} : \begin{array}{c} \text{E}(n) \\ \text{BY GLOBAL SECTIONS} \end{array} \text{ IS GEN' } \right\}$$

IN THIS CASE $\dim(H^0(\mathcal{E}(n))) = 2n + 2$

SUGGESTS UNIF'N



PUTTING BD ON OPEN SUBSET



HECKE OPERATORS

$E/\mathbb{Q}, \sqrt{g} \in E$

$A = C_c^\infty(G(\mathbb{F}) \backslash G(\mathbb{A}) / G(\mathcal{O}), E)$ AUT FORMS

HAS AN ACTION BY HECKE ALG

$C_c^\infty(G(\mathcal{O}) \backslash G(\mathbb{A}) / G(\mathcal{O}), E)$

BY CONVOLUTION (AFTER CHOOSING HAAR MEASURE)

$\cong \bigotimes_{v \in |X|} \mathcal{H}_v, \quad \mathcal{H}_v = C_c^\infty(G(\mathcal{O}_v) \backslash G(\mathbb{F}_v) / G(\mathcal{O}_v), E)$

BY SAITAKE ISOM,

$\mathcal{H}_v \xrightarrow{\sim} \text{REP}(\widehat{G}, E)$

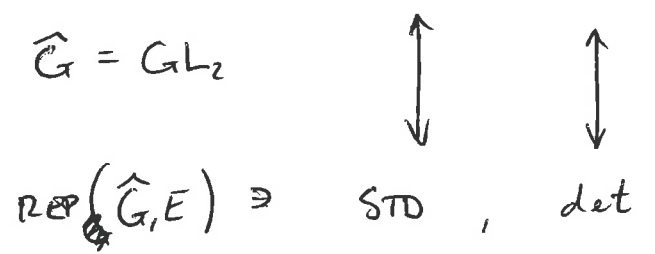
RHS IS PARAM'D BY HIGHEST WTS

$\mu \in X_*(\mathbb{T})_+ = X^*(\widehat{\mathbb{T}})_+$
MINUSULE

$\mathbb{1}_{K \backslash (\mathbb{T}_v)_K} \longleftarrow W_\mu$
(UP TO CONSTANT)
 $K = G(\mathcal{O}_v)$

EX $G = GL_2$, $\pi = \pi_v \in \mathcal{O}_F$ UNIF'R (5)

$\mathcal{H}_v \cong E[T_\pi, \langle \pi \rangle^{\pm 1}]$



HINT TO GEOMETRIZATION:

$GL_n(\mathcal{O}_v) \backslash GL_n(F_v) / GL_n(\mathcal{O}_v) \cong \left\{ (L_1, L_2, i) : \begin{array}{l} L_i \text{ FREE} \\ \mathcal{O}_v\text{-MODS OF} \\ \text{RK } n \end{array} \right\}$

$i: L_1 \otimes F_v \xrightarrow{\sim} L_2 \otimes F_v$

DEFN $Hk(S)$ CLASSIFIES (ξ_1, ξ_2, i, x) , WHERE

"HECKE STACK"

- $\xi_i \in \text{BUN}_G(S)$
- $x \in X(S) \rightsquigarrow \Gamma_x \subset X \times S$ DIVISOR
- $i: \xi_1|_{X \times S \setminus \Gamma_x} \xrightarrow{\sim} \xi_2|_{X \times S \setminus \Gamma_x}$

HAVE MAPS

$Hk \longrightarrow \text{BUN}_G \times \text{BUN}_G \times X$

LET $v \in |X|$, AND LET

$GR_v = \text{FIBER OF } Hk \text{ OVER } \text{BUN}_G \times \{ \text{TRIV} \} \times \{ v \}$

FOR A k -ALG R ,

$$Gr_v(R) = G(R \hat{\otimes} F_v) / G(R \hat{\otimes} O_v)$$

ALSO DENOTED

$$Gr_v = LG / L^*G \quad \text{"AFFINE GRASSMANNIAN"}$$

THIS IS A PROPER IND-Scheme

WANT MORE VARIATION: $I = \text{FINITE SET}$

$$Hk_I \longrightarrow Bun_G \times Bun_G \times X^I \quad \text{"MODIFICATIONS AT MULTIPLE PTS"}$$

Hk_I CLASSIFIES $(\mathcal{E}_1, \mathcal{E}_2, \{x_i\}_{i \in I}, \phi)$, WHERE $\mathcal{E}_1, \mathcal{E}_2, \{x_i\}$ AS BEFORE,

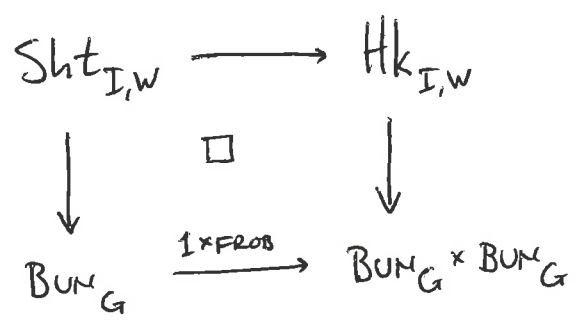
AND

$$\phi: \mathcal{E}_1 \dashrightarrow \mathcal{E}_2, \text{ ISOM AWAY FROM } \Gamma_{x_i}, i \in I$$

HAVE A BOUNDED VERSION OF THIS $Hk_{I,W}$, $W \in \text{REP}_G^I$
"BDDNESS AT Γ_{x_i} BY W_i "

THINK: $Bun_G(k)$ IS FROB-FIXED PT

DEF



$$\text{IE. } \text{Sht}_{I,W}(S) = \{(\mathcal{E}, \{x_i\}_{i \in I}, \phi)\}$$

- $\mathcal{E} = G$ -BUNDLE OVER $X \times_k S$
- $x_i \in X(S)$
- $\phi: (1 \times \text{FROB}_S)^* \mathcal{E} \dashrightarrow \mathcal{E}$, ISOM AWAY FROM Γ_{x_i} , BDD BY W

ASIDE: "BDD BY W"

(7)

$$\Sigma_1 \dashrightarrow \Sigma_2 \quad \text{RK } n \quad \text{v.B.'S } / X$$

FROM THIS, GET ISOM OF VECTOR SPACES

$$\begin{array}{ccc} \Sigma_1|_{F_v} & \xrightarrow{\sim} & \Sigma_2|_{F_v} \\ U & & U \\ L_1 & & L_2 \end{array} \quad \text{LATTICES}$$

ISOM DOES NOT RESPECT LATTICES, BUT CORRESPONDS TO AN ELT OF

$$GL_n(\mathcal{O}_v) \backslash GL_n(F_v) / GL_n(\mathcal{O}_v) = \coprod_{\mu \in X_*(T)_+} [\mu(F_v)]$$

"BDD BY W_μ " = "BDD BY μ IN ORDERING"

IF $I = \emptyset$, $Hk = \text{Bun}_G$, SO WE GET

$$\begin{array}{ccc} \text{Sht}_\emptyset & \longrightarrow & Hk \\ \downarrow & \square & \downarrow \Delta \\ \text{Bun}_G & \xrightarrow{1 \times \text{FRUB.}} & \text{Bun}_G \times \text{Bun}_G \end{array}$$

SO $\text{Sht}_\emptyset = \underline{\text{Bun}_G(k)}$ "CONSTANT SCHEME"

IN GENERAL, $\text{Sht}_{I,W}$ IS A DM STACK