MSRI
Mathematical Sciences Research Institute

17 Gauss Way Berkeley, CA 94720-5070 p: 510.642.0143 f: 510.642.8609 www.msri.org

NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: KAROL KOZIOL Email/Phone: kkoziol@ualberta.ca

Speaker's Name: JARED WEINSTEIN

Talk Title: MODULI OF SHTUKAS I

Date: 4 / 8 / 19 Time: 11 : 00 am / pm (circle one)

Please summarize the lecture in 5 or fewer sentences: THE SPEAKER DISCUSSED SOME OF THE MAIN OBJECTS IN THE GEOMETRIC LANGLANDS PROGRAM: THE STACK OF G-BUNDLES, THE HECKE STACK, AND THE STACK OF SHTUKAS

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

- [x] Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- [x] Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
    - **Computer Presentations**: Obtain a copy of their presentation
    - **Overhead**: Obtain a copy or use the originals and scan them
    - **Blackboard**: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
    - **Handouts**: Obtain copies of and scan all handouts
- [x] For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- [x] When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)
- [x] Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

MODULI OF SHTUKAS I — WEINSTEIN

NUMBER FIELDS

- MODULI OF ELLIPTIC CURVES $E/\mathbb{Q}$
  $E(\mathbb{C}) \cong \mathbb{C}/\Lambda$

∞ ⟷ v

$GL_2/\mathbb{Q}$ STORY

- $\mathbb{G}_m/\mathbb{Q}$ ⟷ CARLITZ MODULES

- CM ELLIPTIC CURVES $/K$ ← IM QUAD'C
  (GL_1/K)

(NO ANALOG... YET)

FUNCTION FIELDS $F = k(X)$

- MODULI OF DRINFELD MODULES OVER $A = H^0(X \setminus |v|, \mathcal{O}_X)$
  C/Λ

- MODULI OF DRINFELD SHTUKAS
  (ONE SIMPLE ZERO / ONE SIMPLE POLE)
  (L. LAFFORGUE FOR GL_n)

- G - SHTUKAS w/ ARBITRARY LEGS

RED GP

- $k = \mathbb{F}_q$, X/k GEOM CONN'D, SMOOTH PROJ CURVE
- $F = k(X)$
- G/k SPLIT RED'VE GP
- Bun_G = STACK WHICH SENDS S/k TO GROUPOID OF G-BUNDLES ON $X \times_k S$ (= G-TORSORS)

  (<u>EX</u> $G = GL_n$ GIVES LOC. FREE $\mathcal{O}$-MODS OF RK n)

FOR $G = GL_n$,

$$\mathrm{Bun}_G(k) = G(F) \backslash G(\mathbb{A}) / G(\mathcal{O}), \qquad G(\mathcal{O}) = \prod_{v \in |X|} G(\mathcal{O}_v)$$

<u>SKETCH</u> EVERY $\mathcal{E} \in \mathrm{Bun}_G(k)$ IS TRIVIAL OVER F:

$$\xi : \mathcal{E}|_U \xrightarrow{\sim} \mathcal{E}^{\mathrm{TRIV}}|_U \qquad U \text{ DENSE OPEN}$$

ALSO OVER EACH $\mathcal{O}_v$:

$$\xi_v : \mathcal{E}|_{\mathcal{O}_v} \xrightarrow{\sim} \mathcal{E}^{\mathrm{TRIV}}|_{\mathcal{O}_v}$$

GET ELEMENT

$$\xi|_{F_v} \circ \xi_v^{-1} \in \mathrm{Aut}\left(\mathcal{E}^{\mathrm{TRIV}}|_{F_v}\right) \cong G(F_v)$$

SO AUT FORMS ARE FNS ON $\mathrm{Bun}_G(k)$

FACT Bun_G IS A SMOOTH ARTIN STACK

$\Longrightarrow$ $\exists$ A SMOOTH UNIFORMIZATION

$$Y \longrightarrow \mathrm{Bun}_G$$

SMOOTH, LOCALLY OF FINITE TYPE (BUT NOT OF FINITE TYPE. IN GENERAL)

EX $\quad \mathrm{Bun}_{GL_1} \cong [\mathrm{Pic}(X)/\mathbb{G}_m] \qquad (X(k) \neq \phi)$

UNIF'N :

$$\coprod_{d \in \mathbb{Z}} \mathrm{Pic}^d(X) \longrightarrow \mathrm{Bun}_{GL_1}$$

DEG d LINE BUNDLES

EX $\quad X = \mathbb{P}^1 \qquad G = GL_2$

$$\mathrm{Bun}_{GL_2} \cong \coprod_{d \in \mathbb{Z}} \mathrm{Bun}^d_{GL_2}$$ DEG d V.B'S

$\mathcal{E} \in \mathrm{Bun}^0_{GL_2}(S)$ ARE EVERYWHERE ISOM'C TO

$$\mathcal{O}(n) \oplus \mathcal{O}(-n), \qquad n \geq 0$$

DEFINE

$$\mathrm{Bun}^{0, \leq n}_{GL_2} = \{ \mathcal{E} : \mathcal{E}(n) \text{ IS GEN'D BY GLOBAL SECTIONS} \}$$

IN THIS CASE $\quad \dim(H^0(\mathcal{E}(n))) = 2n + 2$

SUGGESTS UNIF'N

$$\underbrace{\{ \mathcal{O}^{2n+2} \longrightarrow \mathcal{F} \}}_{\text{QUOT SCHEME}} \longrightarrow \mathrm{Bun}^{0,\,\le n}_{GL_2}$$

RK 2

PUTTING BD ON OPEN SUBSET

$$\mathcal{F} \longmapsto \mathcal{F}(-n)$$

HECKE OPERATORS

$E/\mathbb{Q}$, $\sqrt{q} \in E$

$$\mathcal{A} = C_c^\infty (G(F) \backslash G(\mathbb{A}) / G(\mathcal{O}), E) \qquad \text{AUT FORMS}$$

HAS AN ACTION BY HECKE ALG

$$C_c^\infty (G(\mathcal{O}) \backslash G(\mathbb{A}) / G(\mathcal{O}), E)$$

BY CONVOLUTION (AFTER CHOOSING HAAR MEASURE)

$$\cong \bigotimes_{v \in |X|}{}' \mathcal{H}_v, \qquad \mathcal{H}_v = C_c^\infty (G(\mathcal{O}_v) \backslash G(F_v) / G(\mathcal{O}_v), E)$$

BY SATAKE ISOM,

$$\mathcal{H}_v \xrightarrow{\sim} \mathrm{REP}(\hat{G}, E)$$

$$\mathbb{1}_{K\mu(\pi_v)K} \longleftarrow\!\shortmid \; W_\mu$$

(UP TO CONSTANT)

$K = G(\mathcal{O}_v)$

RHS IS PARAM'D BY HIGHEST WTS

$\mu \in X_*(T)_+ = X^*(\hat{T})_+$

MINUSCULE

EX $G = GL_2$, $\pi = \pi_v \in \mathcal{O}_{F_v}$ UNIF'R

$$\mathcal{H}_v \cong E[T_\pi, \langle \pi \rangle^{\pm 1}]$$

$\widehat{G} = GL_2$

$$\mathrm{REP}(\widehat{G}, E) \ni \mathrm{STD}, \det$$

HINT TO GEOMETRIZATION:

$$GL_n(\mathcal{O}_v) \backslash GL_n(F_v) / GL_n(\mathcal{O}_v) \cong \left\{ (L_1, L_2, i) : \begin{array}{l} L_i \text{ FREE } \mathcal{O}_v\text{-MODS OF RK } n \\ i : L_1 \otimes F_v \xrightarrow{\sim} L_2 \otimes F_v \end{array} \right\} / \cong$$

DEFN $Hk(S)$ CLASSIFIES $(\mathcal{E}_1, \mathcal{E}_2, i, x)$, WHERE

"HECKE STACK"

- $\mathcal{E}_i \in \mathrm{Bun}_G(S)$
- $x \in X(S) \rightsquigarrow \Gamma_x \subset X \times S$ DIVISOR
- $i : \mathcal{E}_1|_{X \times S \setminus \Gamma_x} \xrightarrow{\sim} \mathcal{E}_2|_{X \times S \setminus \Gamma_x}$

HAVE MAPS

$$Hk \longrightarrow \mathrm{Bun}_G \times \mathrm{Bun}_G \times X$$

LET $v \in |X|$, AND LET

Gr_v = FIBER OF Hk OVER $\mathrm{Bun}_G \times \{\mathcal{E}^{\mathrm{triv}}\} \times \{v\}$

FOR A k-ALG R,

$$\mathrm{Gr}_v(R) = G(R \hat{\otimes} F_v) / G(R \hat{\otimes} \mathcal{O}_v)$$

ALSO DENOTED

$$\mathrm{Gr}_v = LG / L^+G \qquad \text{"AFFINE GRASSMANNIAN"}$$

THIS IS A PROPER IND-SCHEME

WANT MORE VARIATION: I = FINITE SET

$$\mathrm{Hk}_I \longrightarrow \mathrm{Bun}_G \times \mathrm{Bun}_G \times X^I \qquad \text{"MODIFICATIONS AT MULTIPLE PTS"}$$

Hk_I CLASSIFIES $(\mathcal{E}_1, \mathcal{E}_2, \{x_i\}_{i \in I}, \phi)$, WHERE $\mathcal{E}_1, \mathcal{E}_2, \{x_i\}$ AS BEFORE,

AND $\phi : \mathcal{E}_1 \dashrightarrow \mathcal{E}_2$, ISOM AWAY FROM Γ_{x_i}, $i \in I$

HAVE A BOUNDED VERSION OF THIS $\mathrm{Hk}_{I,W}$, $W \in \mathrm{Rep}_{\hat{G}^I}$

$\mathrm{Hk}_{I,W} \subset \mathrm{Hk}_I$ "BDDNESS AT Γ_{x_i} BY W_i"

THINK: $\mathrm{Bun}_G(k)$ IS FROB-FIXED PT

DEF

$$\begin{tikzcd}
\mathrm{Sht}_{I,W} \arrow[r] \arrow[d] \arrow[dr, phantom, "\square"] & \mathrm{Hk}_{I,W} \arrow[d] \\
\mathrm{Bun}_G \arrow[r, "1 \times \mathrm{Frob}"] & \mathrm{Bun}_G \times \mathrm{Bun}_G
\end{tikzcd}$$

I.E. $\mathrm{Sht}_{I,W}(S) = \{(\mathcal{E}, \{x_i\}_{i \in I}, \phi)\}$

- $\mathcal{E}$ = G-BUNDLE OVER $X \times_k S$
- $x_i \in X(S)$
- $\phi : (1 \times \mathrm{Frob}_S)^* \mathcal{E} \dashrightarrow \mathcal{E}$, ISOM AWAY FROM Γ_{x_i}, BDD BY W

ASIDE: "BDD BY W"

$$\mathcal{E}_1 \dashrightarrow \mathcal{E}_2 \qquad \text{RK } n \quad \text{V.B.'S } /X$$

FROM THIS, GET ISOM OF VECTOR SPACES

$$\begin{array}{ccc} \mathcal{E}_1|_{F_v} & \xrightarrow{\sim} & \mathcal{E}_2|_{F_v} \\ \cup & & \cup \\ L_1 & & L_2 \end{array} \qquad \text{LATTICES}$$

ISOM DOES NOT RESPECT LATTICES, BUT CORRESPONDS TO AN ELT OF

$$GL_n(O_v)\backslash GL_n(F_v)/GL_n(O_v) = \coprod_{\mu \in X_*(T)_+} [\mu(\pi_v)]$$

"BDD BY W_μ" = "BDD BY μ IN ORDERING"

IF $I = \phi$, $Hk = Bun_G$, SO WE GET

$$\begin{tikzcd} Sht_\phi \arrow[r] \arrow[d] \arrow[dr, phantom, "\square"] & Hk \arrow[d, "\Delta"] \\ Bun_G \arrow[r, "1 \times \text{FROB.}"] & Bun_G \times Bun_G \end{tikzcd}$$

SO $Sht_\phi = \underline{Bun_G(k)}$ "CONSTANT SCHEME"

IN GENERAL, $Sht_{I,W}$ IS A DM STACK