Conjecture: Kaplansky's K - field (can assum K has d elements)

$K[G]$ G - discrete group.

$a, b \in K[G]$, $ab = e$ Then $b \cdot a = e$.

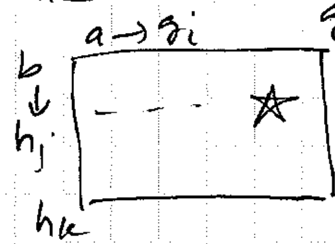
then

$a, b \in K[G]$, $|K| = 2$

$ab = e$

$a = g_1 + \dots + g_n$, $b = h_1 + \dots + h_k$.

$g_i h_j = g_{i'} h_{j'}$



$g_1 h_1 = g_n h_3$

$ba \stackrel{?}{=} e$

If rank $a \leq 5$
& rand $b \leq 7$ $\Rightarrow$ $ab = e$. $ba = e$

2) Conjecture

Connes's - problem.

$B(H)$ — H is Hilbert space

algebra of all bounded operators.

$M \subset B(H)$ is a Neumann? if M is $*$-algebra.

$a \in M \Rightarrow a^* \in M$, M is closed under weak/strong operator.

τ - trace on M, $\tau(1_n) = 1$.

$\tau M \to \mathbb{C}$, $\tau(a) \geq 0$ if $a \geq 0$.

M

$[\lambda(G)] \subset B(l^2(G))$

$\mathbb{C}[\lambda(G)]$ = Von ~~Numann~~ Newman algebra of G. is called.

Group-Com algebra problem $\forall G, N(G) \hookrightarrow R^\omega$.

Theorem:- If G is Sofic Then $N(G) \subset R^\omega$.

Defn Sofic Group.

$d: G \times G \to \mathbb{R}_+$ - distance bi-invariant distance

$d(gx, gy) = d(xg, yg) = d(x, y)$, $\forall x, y, g \in G$.

A sequence of groups (G_i, d_i), where d_i is bi-invariant.

$\mathcal{U}$ - ultrafiller, $\prod G_i$ $\quad N = \{(g_i) \in \prod G_i \mid \lim d(g_i, e) = 0\}$

if d is distance $\Rightarrow N \triangleleft \prod(G_i)$.

$\prod(G_i, d_i) = \prod / N$.

Defn G is Sofic if $G \hookrightarrow \prod_{\mu} (S(n_i), d_{Hamg})$.

Defn G is called hyper linear if $G \xhookrightarrow{\text{embed}} \prod_{\mu} (M(n_i), d)$

$d(u, v) = \operatorname{tr}_n((u-v)^*(u-v))$.

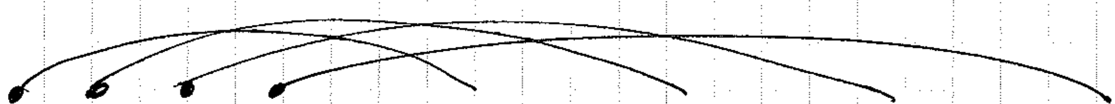
CEP :- $\forall G$ is hyperlinear.
Every Sofic group

3) Gottschalk's surjunctivity conjecture.

A - finite $\quad |A| < \infty \quad G \curvearrowright A^G$,
for every injective G map $\varphi : A^G \to A^G$, φ must be surjective -

4) Fuglede - Kadison - Conjecture.

$\ln \det(\Lambda) \geq 0$ for every positive
$\Lambda \in M_d(\mathbb{Z}(G))$
$\cap \; B(\ell^2(G))$.



Points spread apart -

Def^n Higman Group.

$\langle a, b : b^{-1} a b = a^2 \rangle = BS(1,2)$.

$$\langle a_1, a_2, a_3, a_4 : \begin{aligned} a_2^{-1} a_1 a_2 &= a_1^2 \\ a_3^{-1} a_2 a_3 &= a_2^2 \\ a_4^{-1} a_3 a_4 &= a_3^2 \\ a_1^{-1} a_4 a_1 &= a_4^2 \end{aligned} \rangle$$

"Hig"

Why group is not infinite?

$$\rightarrow BS(1,2) *_{\mathbb{Z}=\langle a_2 \rangle} BS(1,2) > F_2 = \langle a_1, a_3 \rangle$$

$$K_{1,3,4} = \langle a_1, a_3, a_4 : a_4^{-1} a_3 a_4 = a_3^2, \ a_1^{-1} a_4 a_1 = a_4^2 \rangle$$

$$\rightarrow BS *_{\mathbb{Z}=\langle a_4 \rangle} BS > F_2 = \langle a_1, a_3 \rangle.$$

Theorem (Higman): No nontrivial finite

$$Hig \rtimes \mathbb{Z}/4\mathbb{Z} = \langle a, t : t^4 = e, \ (tat^{-1})^{-1} a (tat^{-1}) = a^2 \rangle.$$

If G is amenable then $\forall F_i \subset F_{i+1} \subset G$, s.t.

$\varphi_i F \to S(\cdot)$ is ε-app. of F_i.

Then the action of $\varphi_i(F_i)$ on $\{1, \dots, n_i\}$ is conjugate to the action on disjoint union of Følner sets.

$\{1, 2, \dots, n\}$ $\varphi_i(F_i)$