

NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

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Speaker's Name: Marie-France Vigneras

Talk Title: Simple modules of pro-p Iwahori Hecke algebras

Date: 11 / 21 / 2014 Time: 9 : 30 am / pm (circle one)

List 6-12 key words for the talk: Iwahori Hecke algebra, supersingular representation, reductive p-adic group

Please summarize the lecture in 5 or fewer sentences:
The classification of simple modules over the pro-p Iwahori Hecke algebra is explained. The classification is accomplished by classifying the supersingular modules and then studying their reductions.

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

- Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.
- Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.
 - **Computer Presentations:** Obtain a copy of their presentation
 - **Overhead:** Obtain a copy or use the originals and scan them
 - **Blackboard:** Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
 - **Handouts:** Obtain copies of and scan all handouts
- For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.
- When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list.
(YYYY.MM.DD.TIME.SpeakerLastName)
- Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

F / \mathbb{Q}_p $\mathbb{F}_p((t))$ G red connected / F

$G = \underline{G}(F)$

$T \subseteq B = Z U$

$I = Z_0 T U$

 \uparrow
Iwahorispot $Z = T$

$Z_0 = T_0$

Hecke algebra $(\mathbb{Z}[I_n \backslash G / I_n], \star) = \mathcal{H}$ depends only
on G C alg closed fields of char p Mod C $G \rightarrow$

\downarrow
 $V \neq 0$

Mod $C \cap \mathcal{H}$

\downarrow
 $V \neq 0$

2000-2003 $GL(2, F)$ Irr sq of principal \hookrightarrow Irr $C \cap \mathcal{H}$
seriesIrr C $\mathcal{H}(\bar{F}/F)$ \leftrightarrow other mod
of dim 2 \updownarrow

Irr supercuspidal of

 $GL(2, \mathbb{Q}_p)$ For $GL(n, F)$ Irr $C(\bar{F}/F)$ of dim $n \leftrightarrow$ Irr $C \cap \mathcal{H}$ ~~is not~~ This map is ~~not~~ surjective. $\text{im} \subseteq$ supersingular repsOlivier $\text{im} =$ supersingular reps

H G-split classification of irreducible
 Supersingular representations
 Parabolic induction
 \mathcal{H}_M of G

Abe classified the irreducible non-supersingular representations of H. used parabolic induction from \mathcal{H}_M .

Thm Every ir C rep of H is finite dimensional.

Thm (V) $H = \tilde{H}$ mod its center Z
 $Z = \text{fg. } \mathbb{Z}\text{-alg}$.

$N = N_G(T)$

$N \rightarrow I_1 \setminus G / I_1$

\downarrow
 $N/\mathbb{Z}_M \cong W_M$

$Z_1 \cap W Z_1$

\uparrow
 $(T_w) w \in W_M$

\cup
 $Z/\mathbb{Z}_M = \Lambda_M$

normal subgroup

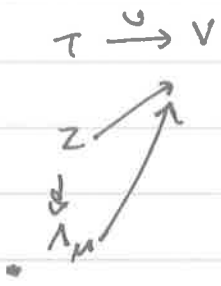
Theorem \mathbb{Z} admits a basis $(3e) e$ - orbits of W_M in Λ_M

T \longrightarrow real vector space dual generated by basis

w valuation of F
 $w(F=0) \cong \mathbb{Z}$

$\Delta(\Gamma, B)$
 $-W_d(t)$





Def A right simple COH module is supersingular if β_C acts by 0 when $\nu(C) \neq 0$.

$$\beta_C = \sum_{\lambda \in C} E(\lambda)$$

"integral" Bernstein elements

$$\mathbb{H}$$

$$E(\lambda) \cdot E(\lambda') = q^{n(\lambda, \lambda')} E(\lambda + \lambda') \quad \lambda, \lambda' \in \Lambda_n$$

$$q = |k_F| \quad n(\lambda, \lambda') \in \mathbb{N} \geq 0$$

$$E(\lambda) \text{ invertible} \iff \nu(\lambda) = 0$$

$$W = \frac{N}{Z_0}$$

U_i
 W^{aff}
 U_i
 S^{aff}

$$W_\mu = \frac{N}{Z_\mu} \rightarrow W \rightarrow 1$$

\uparrow
 $\frac{Z_0}{Z_\mu}$
 \uparrow
 0

\Leftarrow Not a split extension !!

$$W = W^{\text{aff}} \rtimes \Omega$$

fg. commutative

$$W_\mu = W_\mu^{\text{aff}} \cdot \Omega_\mu$$

$$H = H^{\text{aff}} \otimes_{Z[Z_0]} Z[\text{char}] W_\mu = W_\mu^{\text{aff}} \cdot \Omega_\mu^{\text{fg}}$$

comm

6 split / half simple / simply connected

then supersingular simple modules \leftrightarrow ~~characters~~ supersingular characters of $\mathfrak{H}^{\text{aff}} \otimes \mathbb{C}$

Def (χ, σ) is an admissible pair if χ is a character of $Z_K \not\cong \mathbb{Z}$ & $\mathcal{J} \subset S^{\text{aff}}(\chi) = \{s \in S^{\text{aff}} \mid \chi(Z_{K,s}) = 1\}$

$$(U_{\alpha, \beta}, U_{\alpha, \beta, s}) \cap Z_0 \rightarrow Z_{K,s} \begin{matrix} \cap \\ Z_K \end{matrix}$$

It is called supersingular if

$$S^{\text{aff}}(\chi) = S^{\text{aff}} \text{ and } \mathcal{J} = \emptyset$$

(if the root system is irreducible)

Thm character of $\mathfrak{H} \otimes \mathbb{C}$ \leftrightarrow admissible pairs of supersingular \leftrightarrow supersingular

Def (χ, \mathcal{J}, V) an admissible supersingular triple (χ, \mathcal{J}) admissible supersingular pair $\rightsquigarrow \chi$ char of $\mathfrak{H} \otimes \mathbb{C}$

V is a \mathbb{C} -representation of the algebra of \mathfrak{H} in $\Omega_{\mathfrak{H}}$.

$$\text{Irr}(\chi, \mathcal{J}, V) = (\mathfrak{H} \ltimes V) \otimes_{\mathfrak{H} \otimes \mathbb{C}} \mathbb{C}[a_{\alpha}^{\pm}] \mathfrak{H}.$$

Thm: I_M supersingular H -modules

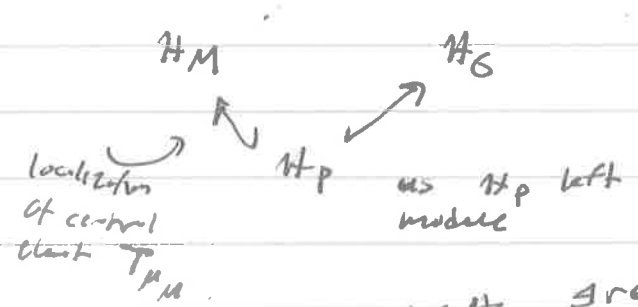


admissible supersingular types

$I(X, J, V) \longleftrightarrow \mathcal{I}(X, J, V)$

$B \subseteq P \subset G$
 \uparrow
 $M \subseteq B$

$M \subseteq P \subset G$



$u \in H_G$ $\Delta \in \mathbb{N}$
 \cdot st $T_{M, H}^\Delta$
 \uparrow
 $\oplus H_P T_{M, H}^\Delta$
 $u \in W_{0, M} \setminus W_{0, G}$

I_P^G : module \longrightarrow mod H_G

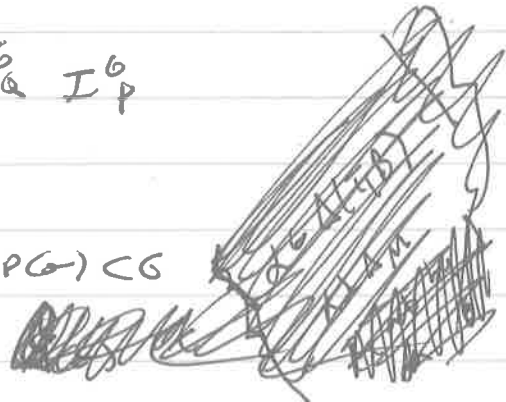
$M \longmapsto M \otimes_{H_P} H_G$

Thm I_P^G is faithful and has left + right adjoints. Transition

$P \subset Q \subset G \quad I_P^G = I_Q^G \circ I_P^Q$

$P \subseteq M \subseteq B \quad \Delta_M \subset \Delta(Q, B)$

$\sigma = \text{FM}$ s.s. H -module $M \quad P \subset P(G) \subset G$



$$P(\sigma) \Leftrightarrow \Delta(\sigma) = \left\{ \begin{array}{l} \lambda \in \Delta(T, B) \\ \text{null} \end{array} \right\} \left. \begin{array}{l} \lambda \perp \Delta M \\ \text{Tracks by } 1 \text{ or } \sigma \\ \text{for } \lambda \text{ image} \\ \text{in } \Delta M \text{ of} \\ \mathbb{Z} \cap (\cup_{i=1}^n u_i) \end{array} \right\}$$

Def (P, σ, Q) is an admissible triple if $B \subset P$, σ inv s.s.

rep of M , $P \subset Q \subset P(\sigma)$
 $A(\sigma) \rightarrow B$

$I(P, \sigma, Q) = I_{P(\sigma)}^{\sigma}$ st Q (σ)
 rep of M ↑ generalization
Sturberg

$M \dots \rightarrow M_Q$

σ extends $e(\sigma)$ $Q \cap N(\sigma)$

st Q $= I_{Q \cap N(\sigma)}^{\sigma} e(\sigma)$

$\sum_{Q \subset Q' \subset P(\sigma)} I_{Q'}^{\sigma} e(\sigma)$

Thm (Abc) For rep of $\mathbb{C} \otimes H$



Admissibility ~~is~~

$$(p, \sigma, q) \Leftrightarrow I(p, \sigma, q)$$