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NOTETAKER CHECKLIST FORM

(Complete one for each talk.)

Name: Justin Hilburn **Email/Phone:** jhilburn@uoregon.edu

Speaker's Name: Marie-France Vigneras

Talk Title: Simple modules of pro-p Iwahori Hecke algebras

Date: 11 / 21 / 2014 **Time:** 9 : 30 am / pm (circle one)

List 6-12 key words for the talk: Iwahori Hecke algebra, supersingular representation, reductive p-adic group

Please summarize the lecture in 5 or fewer sentences: The classification of simple modules over the pro-p Iwahori Hecke algebra is explained. The classification is accomplished by classyfing the supersingular modules and then studying their reductions.

CHECK LIST

(This is **NOT** optional, we will **not pay** for **incomplete** forms)

☑ Introduce yourself to the speaker prior to the talk. Tell them that you will be the note taker, and that you will need to make copies of their notes and materials, if any.

☑ Obtain ALL presentation materials from speaker. This can be done before the talk is to begin or after the talk; please make arrangements with the speaker as to when you can do this. You may scan and send materials as a .pdf to yourself using the scanner on the 3rd floor.

- **Computer Presentations**: Obtain a copy of their presentation
- **Overhead**: Obtain a copy or use the originals and scan them
- **Blackboard**: Take blackboard notes in black or blue **PEN**. We will **NOT** accept notes in pencil or in colored ink other than black or blue.
- **Handouts**: Obtain copies of and scan all handouts

☑ For each talk, all materials must be saved in a single .pdf and named according to the naming convention on the "Materials Received" check list. To do this, compile all materials for a specific talk into one stack with this completed sheet on top and insert face up into the tray on the top of the scanner. Proceed to scan and email the file to yourself. Do this for the materials from each talk.

☑ When you have emailed all files to yourself, please save and re-name each file according to the naming convention listed below the talk title on the "Materials Received" check list. (*YYYY.MM.DD.TIME.SpeakerLastName*)

☑ Email the re-named files to notes@msri.org with the workshop name and your name in the subject line.

$F / \mathbb{Q}_p$ $\qquad$ $\underline{G}$ red connected $/F$

$\mathbb{F}_p((t))$ $\qquad$ $G = \underline{G}(F)$

$T \subseteq B = Z U$

$I = Z_0 I_u$

Iwahori

split $Z = T$ $\qquad$ $Z_0 = T_0$

Hecke algebra $(\mathbb{Z}[I_u \backslash G / I_u], *) = \mathcal{H}$ $\qquad$ depends only on G

C alg closed field of char p

$\mathrm{Mod}_C\, G \longrightarrow \mathrm{Mod}_{C \otimes \mathcal{H}}$

$V \neq 0 \qquad V^{I_u} \neq 0$

2000-2003 $GL(2, F)$

Irr sq of principal series $\hookrightarrow$ Irr $C \otimes \mathcal{H}$

$\mathrm{Irr}_C\, \mathrm{Gal}(\bar{F}/F)$ of dim 2 $\longleftrightarrow$ other irred

$\updownarrow$

Irr supercuspidal of $GL(2, \mathbb{Q}_p)$.

For $GL(n, F)$

$\mathrm{Irr}_C(\bar{F}/F)$ of dim n $\hookrightarrow$ $\mathrm{Irr}_{C \otimes \mathcal{H}}$

This map is ~~not~~ surjective.

$\mathrm{im} \subseteq$ supersingular reps

Ollivier $\quad \mathrm{im} =$ supersingular reps

$\mathcal{H}$ G-split classification of irreducible supersingular representation
Parabolic induction
$\mathcal{H}_M$ of G

Abe classified the irreducible non-supersingular representations of $\mathcal{H}$. used parabolic induction from $\mathcal{H}_M$.

<u>Thm</u> Every irr C rep of $\mathcal{H}$ is finite dimensional.

<u>Thm (V)</u> $\mathcal{H}$ = f.g. module over its center Z
Z = f.g. $\mathbb{Z}$-algebra.

$N = N_G(T)$. $N \to I_1 \backslash G / I_1$

$$\begin{array}{l} N \\ \downarrow \\ N/Z_M = W_M \\ \cup\!\!\mid \\ Z/Z_M = \Lambda_M \\ \text{normal subgroup} \end{array}$$

$I_1 n_w I_1$
$\updownarrow$
(T_w) $w \in W_M$

<u>Theorem</u> Z admits a basis $(z_e)_e$ = orbits of W_M in Λ_M

$T \longrightarrow$ real vector space dual generated by basis $\Delta(T, B)$

$-W\alpha(t)$

$t \longmapsto$

W valuation of F

$w(F-0) \mathbb{Z}$

$$\begin{tikzcd} T \arrow[r, "\nu"] & V \\ Z \arrow[ur] \arrow[d] & \\ \Lambda_\mu \arrow[uur] & \end{tikzcd}$$

Def A right simple $C \otimes \mathcal{H}$ module is supersingular iff $\mathfrak{z}_C$ acts by 0 when $\nu(C) \neq 0$.

$$\mathfrak{z}_C = \sum_{\lambda \in C} E(\lambda)$$

"integral" Bernstein elements

$$\cap$$
$$\mathcal{H}$$

$$E(\lambda) \cdot E(\lambda') = q^{n(\lambda, \lambda')} E(\lambda\lambda') \qquad \lambda, \lambda' \in \Lambda_\mu$$

$$q = |k_F| \qquad n(\lambda, \lambda') \in \mathbb{N}_{\geq 0}$$

$$E(\lambda) \text{ invertible} \iff \nu(\lambda) = 0$$

$$W = \frac{N}{Z_0} \qquad W_\mu = \frac{N}{Z_\mu} \to W \to 1$$

$$\cup\!|$$
$$W^{aff}$$
$$\cup\!|$$
$$S^{aff}$$

$$\uparrow$$
$$\frac{Z_0}{Z_\mu}$$
$$\uparrow$$
$$0$$

$\Leftarrow$ Not a split extension !!

$$W = W^{aff} \rtimes \Omega$$

f.g. commutative

$$W_\mu = W_\mu^{aff} \underset{Z_K}{\cdot} \Omega_\mu$$

$$\mathcal{H} = \mathcal{H}^{aff} \tilde{\otimes}_{\mathbb{Z}[Z_K]} \mathbb{Z}[\Omega_\mu]^{W_\mu} = W_\mu^{aff} \underset{Z_K}{\cdot} \Omega_\mu^{(\nu)} \qquad \text{comm}$$

G split, half simple, simply connected

<u>Thm</u> supersingular simple modules of $\mathcal{H}^{\mathrm{aff}} \otimes \mathbb{C}$ $\longleftrightarrow$ supersingular characters of $\mathcal{H}^{\mathrm{aff}} \otimes \mathbb{C}$

<u>Def</u> (χ, J) is an admissible pair if χ is a character of Z_K & $J < S^{\mathrm{aff}}(\chi) = \{ s \in S^{\mathrm{aff}} \ \chi(Z_{K,s}) = 1 \}$

$$(U_{\alpha A}, U_{-\alpha_s}) \cap Z_0 \twoheadrightarrow Z_{K,s} \subset Z_K$$

It is called supersingular if
$S^{\mathrm{aff}}(\chi) = S^{\mathrm{aff}}$ and $J = \emptyset$
(if the root system is irreducable)

<u>Thm</u> characters of $\mathbb{C} \otimes \mathcal{H}$ $\longleftrightarrow$ admissible pairs

$\cup$ supersingular $\quad$ $\cup$ supersingular

<u>Def</u> (χ, J, V) an admissible supersingular triple (χ, J) admissible supersingular pair $\mapsto \xi$ char of $\mathcal{H}^{\mathrm{aff}}$

V = irr $\mathbb{C}$-representation of the Rstabilizer of ξ in Ω_1.

$$\mathrm{Irr}(\chi, J, V) = (\xi \otimes V) \otimes_{\mathcal{H}^{\mathrm{aff}}} \mathbb{C}[\Omega_1^{?}] \ \mathcal{H}.$$

Thm: Irr supersingular $\mathcal{H}$-modules

$\longleftrightarrow$

admissible supersingular triples

$$I(\chi, J, V) \longleftrightarrow (\chi, J, V)$$

$B \subseteq P \subset G$ $\quad$ $M \subset P \subset Q \subset G$

MB

$\mathcal{H}_M \quad \mathcal{H}_G$

$\mathcal{H}_P$

localization of central elmt T_{μ_M}

as $\mathcal{H}_P$ left module

$h \in \mathcal{H}_G \quad \exists r \in \mathbb{N}$

st $T_{\mu_M}^r h$

$\in$

$\bigoplus \mathcal{H}_P T_{\tilde{w}}$

$w \in W_{0,M} \backslash W_{0,G}$

$$I_P^G : \operatorname{Mod} \mathcal{H}_M \longrightarrow \operatorname{Mod} \mathcal{H}_G$$

$$m \longmapsto m \otimes_{\mathcal{H}_P} \mathcal{H}_G$$

Thm I_P^G is faithful and has left + right adjoints. Transitive

$$P \subset Q \subset G \quad I_P^G = I_Q^G \, I_P^G$$

$P = MB$ $\quad$ $\Delta_M \overset{\Delta(T, B \cap M)}{\overset{\shortparallel}{}} \subset \Delta(T, B)$

σ = Irr s.s. $\mathcal{H}_M$-module w/ $P < P(\sigma) < G$

75

$$P(\sigma) \leftrightarrow \Delta(\sigma) = \left\{ \alpha \in \Delta(T, B) \;\middle|\; \begin{array}{l} \alpha \perp \Delta_M \\ T_\lambda \text{ acts by } 1 \text{ on } \sigma \\ \text{for } \lambda \text{ image} \\ \text{in } \Lambda_\mu \text{ of} \\ \mathbb{Z} \cap (u_\alpha, u_{-\alpha}) \end{array} \right\}$$

$\Delta_M \sqcup$

Def (P, σ, Q) is an admissible triple if $B \subset P$, σ irr s.s.
$\cap B$

rep of H_M, $P \subset Q \subset P(\sigma)$
$\Delta(\sigma) \supset B$

$$I(P, \sigma, Q) = I^G_{P(\sigma)} \; St^{n(\sigma)}_Q \; (\sigma)$$

rep of H_G — Generalized Steinberg

$H_M \quad \cdots \quad H_{M_Q}$

σ extends $e(\sigma)$ $\quad Q \cap n(\sigma)$

$$St^{n(\sigma)}_Q = \frac{I^{\pi(\sigma)}_{Q \cap n(\sigma)} \; e(\sigma)}{\sum_{Q \subsetneq Q' \subset P(\sigma)} I^{n(\sigma)} \; e(\sigma)}$$

<u>Thm</u> (Abe) For rep of $\mathbb{C} \otimes H$

$\longleftrightarrow$

Admissible ~~#~~

$(P, \sigma, Q) \longleftrightarrow I(P, \sigma, Q)$