Spanning trees of graphs on surfaces and the intensity of loop-erased random walk on Z^2

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We show how to compute the probabilities of various connection topologies for uniformly random spanning trees on graphs embedded in surfaces. As an application, we show how to compute the "intensity" of the loop-erased random walk in Z^2, that is, the probability that the walk from (0,0) to infinity passes through a given vertex or edge. For example, the probability that it passes through (1,0) is 5/16; this confirms a 15-year old conjecture about the stationary sandpile density on Z^2. We do the analogous computation for the triangular lattice, honeycomb lattice and Z x R, for which the probabilities are 5/18, 13/36, and 1/4-1/\pi^2 respectively.

(Joint work with Rick Kenyon.)