Are the laws of entanglement thermodynamical?

M. H., J. Oppenheim and R. Horodecki, quant-ph/0207.......

M. H., K. H, P.H, R.H, J. Oppenheim, A. Sen(De), U. Sen quant-ph/0207168

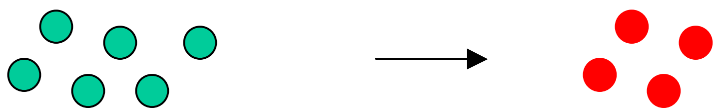
Gdańsk

Supported by European Commision
EQUIP, IST-11053-1999

State transformations in asymptotic regime

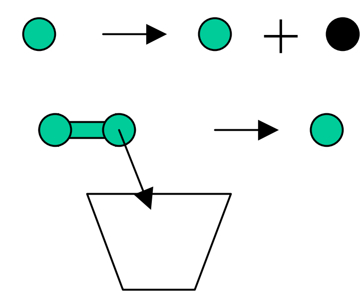
- is it posible to transform ❐ state into ♦ ?
- if so, what is transition rate ?

Transformation ❐ ✹ ♦

❐ ➷n  ❐ out ≅ ♦ ➷m

Class of allowed operations **C**, including usually

- adding a system in state **from some set** 🌢
- removing a subsystem



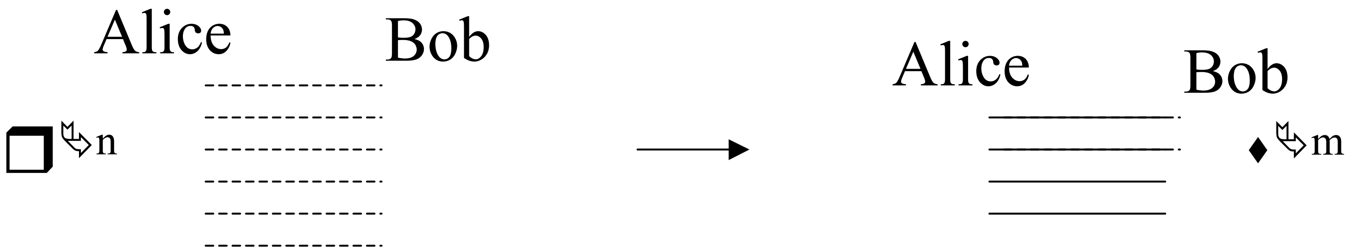
Rate of transition:

$$R = \lim \frac{\text{number of } \textbf{output} \text{ copies}}{\text{number of } \textbf{input} \text{ copies}} = \lim \frac{m}{n}$$

Optimal rate of transition: R(❐ ✹ ♦)

State transformations in entanglement theory

Bennett, Bernstein, Brassard, DiVincenzo Popescu, Schumacher, Smolin, Wootters (1996-97)



Allowed operations **LOCC** (local ops. and classical communication):

- adding locally a system in any state
- removing a local subsystem
- local unitary transformation
- communication through *dephasing channel*

$$\Psi_+ = 1/\sqrt{2}\,(|00\rangle + |11\rangle)$$ (**SINGLET** state)

Optimal rate of transition:

$R(\rho \to \Psi_+)$ is called **entanglement of distillation** E_D

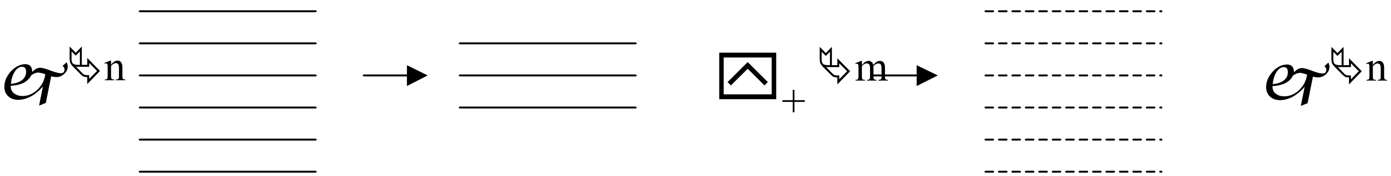
$1/R(\Psi_+ \to \rho)$ is called **entanglement of cost** E_C

Separable (disentangled) states

1) No singlets needed to create state: $E_C = 0$

2) No singlets can be drawn from state: $E_D = 0$

Pure entangled states: reversibility

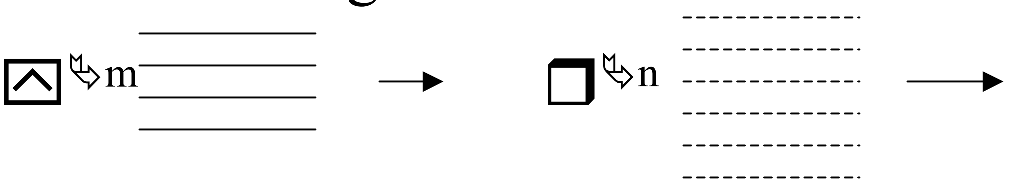


$E_D = E_C = S(\rho_A)$

Mixed states: irreversibility in entanglement theory

Bound entangled states:

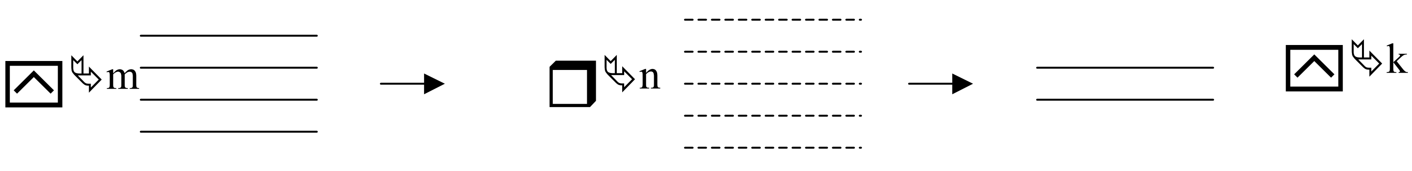
(MH, PH, RH 1998)



No pure entangled states

Generic mixed state:

Rigorously: Dur,Cirac Vidal 2000)



$k < n$

$E_D < E_C$

Thermodynamics and entanglement: first attmepts

Mixed states:

Idea: irreversibility in mixed states distillation is something like irreversibility expressed by II law of thermodynamics

P. Horodecki, M.H., R Horodecki
Acta Phys Slovaca 1998

entanglement - energy
mixed states - heat

bound entanglement - bound energy

In thermodynamics there are both reversibility and irreversibility, hence there should be place for analogy.

Pure states:

Idea: reversibility in pure states transformations is like Carnot cycle.

S. Popescu and D. Rohrlich, PRA 1997

M. Plenio and V. Vedral, PRA 1998

(V. Vedral and E. Kashefi PRL2002)

↓

Mixed-state entanglement is not thermodynamical-like, because there is irreversibility.

Phenomenological thermodynamics (for children)

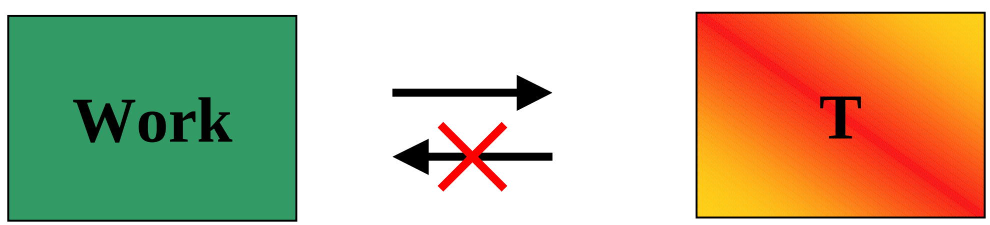
There are **TWO FORMS** of energy:

- ordered one (e.g. mechanical energy)
- disordered one (heat)

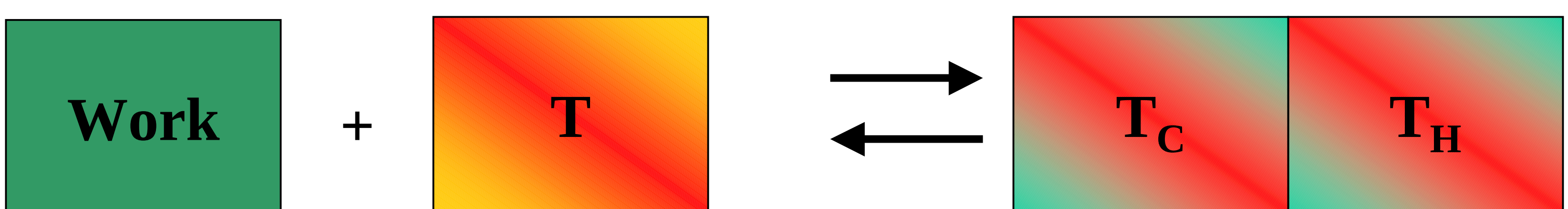
IRREVERSIBLE

One CAN change into (Joule's experiment, FIRST LAW)

One CANNOT change into (SECOND LAW)



One can **DILUTE** into reversibly (CARNOT CYCLE)



Candidate for thermodynamics of entanglement

entanglement - energy

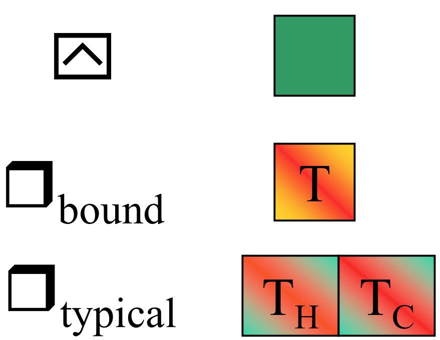
TWO FORMS of entanglement:

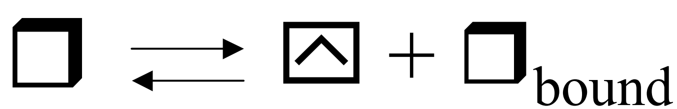
ORDERED: **PURE-STATE** entanglement (mechanical energy)

DISORDERED: **BOUND** entanglement - (heat)

Question: What about typical mixed state (not pure and not bound) ???

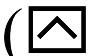
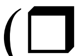
Answer: Typical mixed state is bound entanglement diluted into pure one

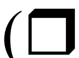




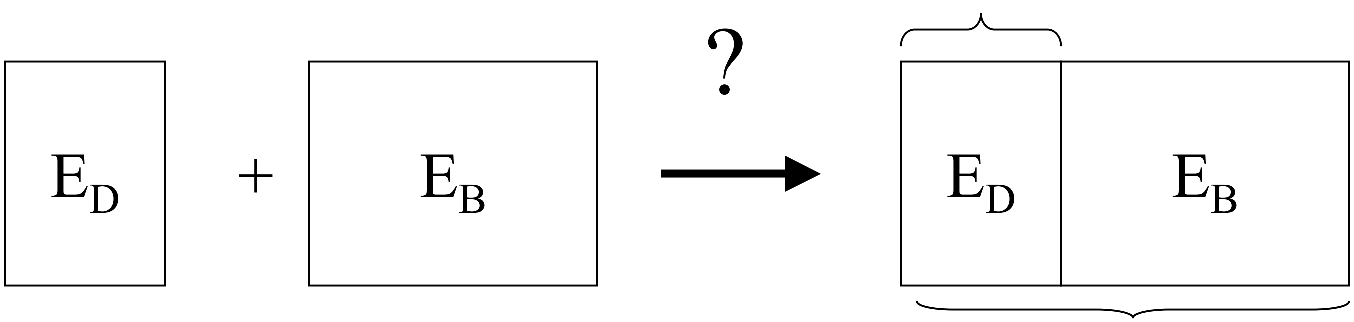
REVERSIBLE DILUTION

Test for thermodynamics of entanglement

E_C() + E_C ( bound) = E_C ()

- total entanglement is conserved
- E_C() is diluted, and it becomes E_D ()

Useful entanglement
(entanglement of dist. E_D)



Total entanglement
(entanglement cost E_C)

Obstacle: perhaps sometimes ❐$_{bound}$ ⇘ ♦ $_{bound}$ is no longer bound ent.

(Shor , Smolin, Terhal 199..)

Proposed solution: instead of bound ent. states, consider:

The largest (nontrivial) set closed under tensor product and LOCC operations - HyperSet

Conjecture: HyperSet exists, and it is the set of PPT states.

Definition: PPT-entanglement cost is number of singlets needed to create ❐ by LOCC if one can add for free PPT states.

TEST: For all statates PPT-entanglement cost should be equal to distillable entanglement.

Partially proven counterexample and example

Counterexample:

$$\rho = p\,|\Phi_+\rangle\langle\Phi_+| + (1-p)\,|\Phi_-\rangle\langle\Phi_-|$$

$$\Phi_+ = 1/\sqrt{2}\,(|00\rangle + |11\rangle)$$

$$\Phi_- = 1/\sqrt{2}\,(|00\rangle - |11\rangle)$$

Vidal, Cirac,Dur 200.. (building on Bennett, DiVincenzo,Smolin and Wootters 1997)

$$E_C = H[1/2+\sqrt{p(1-p)}]$$

Rains 1999, building on Bennett, DiVincenzo, Smolin and Wootters, 1997 and Plenio and Vedral 1998

$$E_D = 1-H(p)$$

$\rightarrow$ $E_C > E_D$

Under some conjecture: $E_C^{PPT} = E_C$

$E_C^{PPT} > E_D$

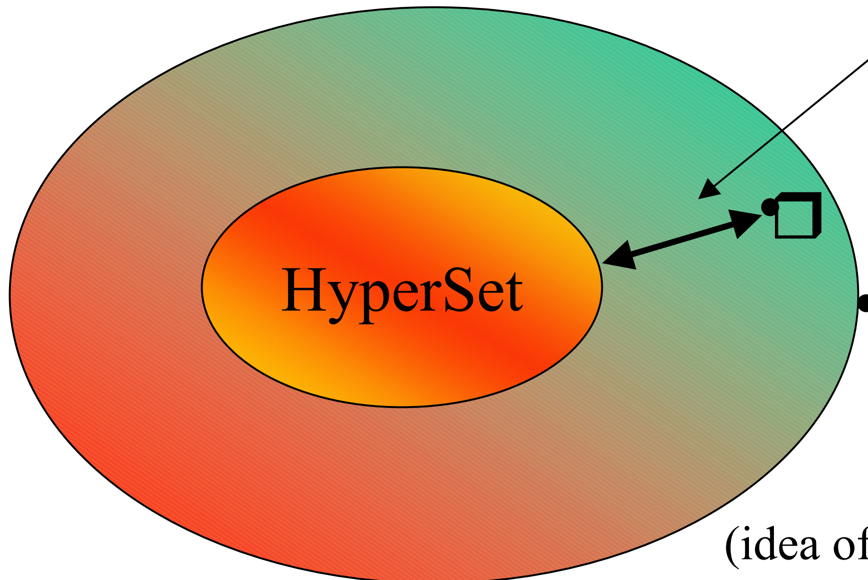
Example: Eisert, Adenauert and Plenio (July 2002) showed that for Werner states

$$E_D^{PPT\text{-}Maps} = E_C^{PPT\text{-}Maps}$$

Some general considerations

C - class of operations (typically convex, closed under tensor product)

HyperSet - the largest set closed under C and tensor product



R(❒✹ ❒$_{Ref}$)

❒$_{Ref}$ -reference state

(idea of relative entropy distance:
- in large deviation theory
- in entangelment theory

Vedral, Plenio, Rippin and Knight 1997

Vedral and Plenio 1998)

RelDis(❒) = inf$_{♦☜HyperSet}$ S(❒|♦)

❒$_{Ref}$ - such that RelDis(❒$_{Ref}$) =1

„**Proposition**”: If the transition ❒✹ ❒$_{Ref}$ is **reversible** then the transition rate R(❒✹ ❒$_{Ref}$) is equal to (regularized) relative entropy distance

R(❒✹ ❒$_{Ref}$) = RelDis(❒)

„Information” model

Class of operations: Noisy Operations (NO)

1) adding ancilla in maximally mixed state
2) tracing out subsystem
3) unitary transformations

HyperSet: consists of maximally mixed state

Reference state: one-qubit pure state

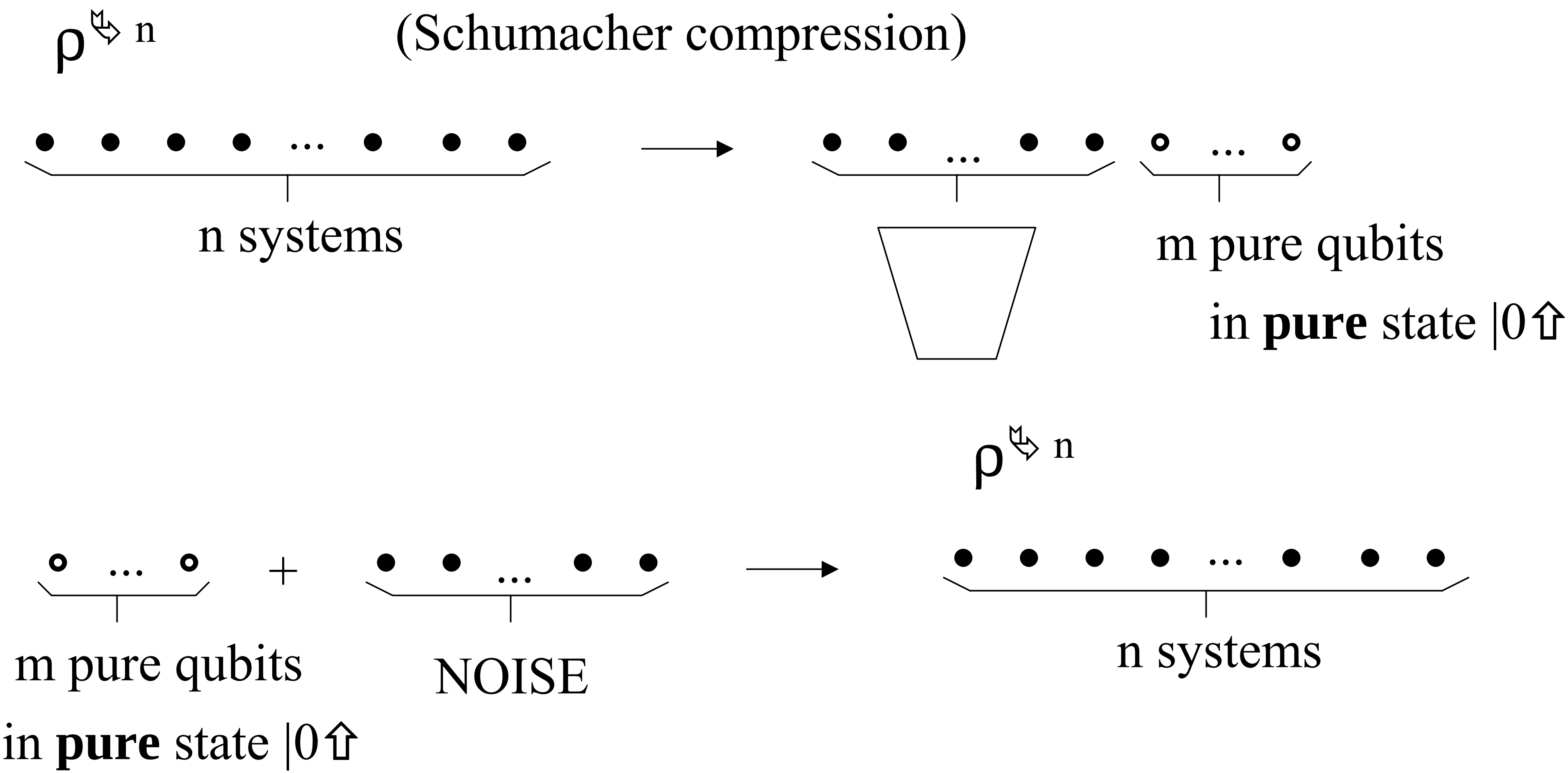
RelDis($❐_{Ref}$) = S($❐_{Ref}$|MaxMixedState)= 1-S($❐_{Ref}$)= 1

For state on d dimensional Hilbert space we have

R(❐✹ $❐_{Ref}$) = log d - S(❐) = RelDis(❐)

(follows from compression theorem)

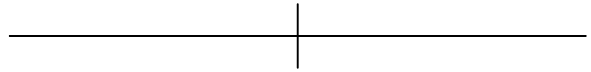
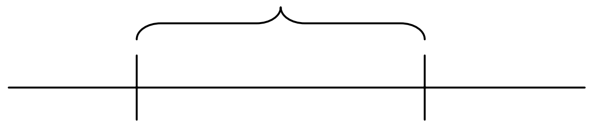
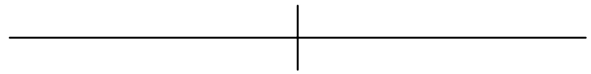
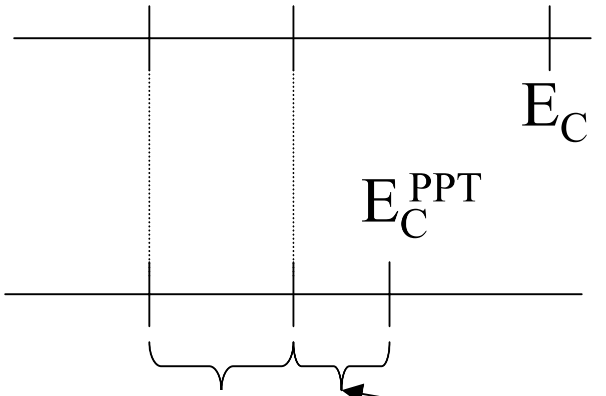
Reversibility in „information" model



Conclusions:

1) Mixed state does not imply irreversibility

2) to form mixed state reversibly, NOISE is needed

TYPE OF STATE	Graphic presentation	TYPE OF AXIOMS:
Pure states	$E_D = E_C = \mathrm{RelDis}$ 	„information as a resource”
Mixed states (2)	E_{bound}  $E_D = \mathrm{RelDis}$ E_C $E_D = E_C^{PPT} = \mathrm{RelDis}$ 	„reversible thermodynamics”
Mixed states (1)	E_D RelDis E_C E_C^{PPT} 	„thermodynamics with phase transitions” ???

„Dissipation’’ of distillation?

„Dissipation’’ of formation?

Conclusions

- Thermodynamical analogy „entanglement-energy" can be tested
- most probably:
    - in general it does not hold
    - for some classes of states it holds
- even if it does not hold, it may be useful: a part of some reacher picture

- scheme of reversible transformations with mixed states was presented
- irreversibility in entanglement processing is not due to mixed states

Soon : feedback to thermodynamics - expressing thermodynamical work in terms of RelDis to set of states out of which no work can be drawn

Open problems:
- to find HyperSet
- to prove asymptotic continuity and monotonicity of *PPT-entanglement of formation* (conjectures needed to counterexample)